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ACTIONS OF COMPACT GROUPS ON AF C^*-ALGEBRAS[1]

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Introduction

Let G be a compact group, and $\bar{A}$ an AF (approximately finite dimensional) C^* algebra. Suppose α: $G \to \mathrm{Aut}(\bar{A})$ is a point norm continuous group homomorphism such that (i) $\alpha(G)$ leaves globally invariant a dense locally finite dimensional *-subalgebra; and (ii) $\alpha(G)$ is locally representable (i.e., there exists a dense locally finite dimensional *-subalgebra $A = \bigcup A_i$, with A_i finite dimensional and invariant, *and* the action of α restricted to each A_i arises from a representation of G in the unitary group of A_i). Then a complete invariant for conjugacy in $\mathrm{Aut}(\bar{A})$ of α may be given in terms of a suitable partially ordered module over the representation ring of G, together with a specified positive element.

This classification applies (as a very special case) to product type actions on UHF algebras, and also to groups of prime order of approximately inner automorphisms acting on arbitrary AF algebras, but satisfying (i) (§IV). Moreover, the invariant can be used to construct weird actions, which are certainly not of product type, on UHF algebras, for any compact group (§III, VI).

We also show that if $\bar{A}$ has unique trace, then the complex vector space generated by the traces of $\bar{A} \times_\alpha G$ (the crossed product) is a cyclic module over the complexified representation ring of G, when G is finite. This is true for any unital C^* algebra $\bar{A}$ with unique trace, and any action of G, and is obtained by directly constructing all the traces on $R \times_\alpha G$, where R is the tracial completion of $\bar{A}$.

Our emphasis here is on the crosssed product $\bar{A} \times_\alpha G$ (as opposed to the fixed point algebra $\bar{A}^G$ studied in [11]). The Grothendieck group (K_0) of this admits a natural ordered module structure over the representation ring of G. The first key result, II.1 and II.2, describe the ordered module as a limit of finitely generated ones (this is precisely analogous to dimension groups arising as limits of simplicial groups; here $\mathbf{Z}$, the ordered ring, is replaced by the

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