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# EQUIVARIANT ISOTOPIES AND SUBMERSIONS

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## Introduction

Lees' topological immersion theory [1] has been generalized in two ways. In [2] an equivariant immersion theory was developed, and in [3] Gauld developed a submersion theory. These theories have been important in smoothing theory, and for similar reasons it is desirable to have an equivariant submersion theory for the topological category. (A smooth equivariant submersion theory, in fact a smooth equivariant Gromov theory has already been given by Bierstone [4].)

By now the form of such arguements is routine. The key result needed is an equivariant lifting theorem for submersions (Theorem A below). As in [2], we would like to use Siebenmann's deformation of stratified spaces theorem [5] to derive this. In fact, Siebenmann shows that his theorem gives a (non-equivariant) lifting theorem for submersions; and indeed, the same argument would generalize to equivariant submersions with trivial $G$ action on the target space. The problem is that one needs a $G$ isotopy extension theorem in which the parameter space has a non-trivial $G$ action. As is often the case in equivariant theories, this problem is solved by reducing it to the case that the parameter space has a single orbit type. (See the proof of the Fibrewise $G$ deformation theorem in Section 3.) Finally, in trying to follow Gauld's proof of the lifting theorem, it is necessary to understand intersections of equivariant tubes and products (3.1 and Corollary 3, Section 3).

DEFINITION. A *$G$-manifold $M^n$* is a second countable Hausdorff $G$-space $M$ such that for each $x \in M$ there is an $n$-dimensional $G_x$ orthogonal representation space $V_x$ and a $G_x$ homeomorphism $h_x$ of $V_x$ onto a neighborhood of $x$ with $h_x(0) = x$, $G_x$ the isotropy subgroup of $x$. We call $h_x$ a *$G_x$ chart*. Because $G$ is finite, this is equivalent to Bredon's notion [6] of a locally smooth $G$-manifold.

DEFINITION. Let $N$ and $Q$ be $G$ manifolds. A $G$ map $f\colon N \to Q$ is a $G$ submersion if for each $x \in N$ we can find a $G_x$ chart $h_x\colon V_x \to N$ and a $G_y$

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