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SOME RESULTS ON CAUCHY SURFACE CRITERIA IN LORENTZIAN GEOMETRY

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1. Introduction

Let M denote an arbitrary *space-time*, by which we mean a smooth connected time oriented Lorentzian manifold of dimension $n \geq 2$ having signature $(- + \cdots +)$. Causality conditions have played an important role in the development of the global theory of Lorentzian geometry (e.g., see Beem and Ehrlich [1]). One such condition which, for instance, ensures the geodesic connectivity of causally related points in M is *global hyperbolicity*. (For definitions and basic results in the causal theory of space-time, see, for example, Penrose [8] or Hawking and Ellis [6]. For a short exposition of the subject we recommend the excellent review article by Geroch and Horowitz [5].) A classical theorem of the causal theory due to Geroch [4] says that a space-time M is globally hyperbolic if and only if it admits a Cauchy surface. (A subset S of M is a Cauchy surface if and only if each inextendible timelike curve in M intersects S once and only once. A Cauchy surface for M is necessarily a codimension one topological submanifold of M.) In this paper we present a general result which establishes necessary and sufficient conditions on a subset S of M to be Cauchy. Its advantage over a related result of Geroch [4] (which is discussed in the next section) is that it does *not* require that S be *achronal* (i.e., that each timelike curve in M intersect S *at most* once). This general result is then used to obtain Cauchy criteria in more specific situations. In particular we obtain a technical improvement of the result of Budic et. al. [2] that a C^1 spacelike hypersurface S in a globally hyperbolic space-time is necessarily Cauchy. (Besides its use in the definition of a spacelike hypersurface, the C^1 differentiability assumption is used in their proof at one point to invoke the inverse function theorem.) Our version removes in a natural way the differentiability assumption on S (and, in particular, requires a weakening of the notion of "spacelike").

We also obtain a result concerning the topological structure of a certain class of space-times M. If S is a Cauchy surface for M then, as is well known, M must be homeomorphic to $\mathbf{R} \times S$. Here we determine the topology of those space-times M admitting a hypersurface S which is *not* Cauchy but which satisfies a certain subset of our general Cauchy criteria.

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