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WASHNITZER'S CONJECTURE AND THE COHOMOLOGY OF A VARIETY WITH A SINGLE ISOLATED SINGULARITY

BY

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Introduction

Let X be an irreducible quasi-projective variety defined over $\mathbf{C}$, the field of complex numbers, and let $H^*(X, \mathbf{C})$ denote the singular cohomology.

One has a morphism of sites π: $X_{\text{Class}} \to X_{\text{Zar}}$, hence a Leray spectral sequence

$$H^p_{\text{Zar}}(X, R^q_{\pi_*}\mathbf{C}) \to H^{p+q}(X, \mathbf{C}),$$

which yields a decreasing filtration in $H^{p+q}(X, \mathbf{C})$. Washnitzer conjectured that if X is non-singular the filtration above coincides with the filtration by "coniveau". Recall that this filtration, also called the arithmetic filtration, is given by

$$N^pH^m = \cup \operatorname{Ker}\{H^m(X) \to H^m(X - Z): Z \text{ is Zariski closed and } \operatorname{cod} Z \geq p\}.$$

Bloch and Ogus proved Washnitzer's conjecture in [2].

We extend their results to the case of a variety with at most a single isolated singularity.

We fix a distinguished closed point x_0 on X and assume that $X - \{x_0\}$ is non-singular. In this case we say that X is almost non-singular [3].

We define $N^{+0} = H^m$ and for $p \geq 1$

$$N^{+p}H^m = \cup \operatorname{Ker}\{H^m(X) \to H^m(X - Z): Z \text{ is Zariski closed, } \operatorname{cod} Z \geq p \text{ and } x_0 \notin Z\}.$$

Our result is that this arithmetic filtration coincides with the Leray filtration induced by the morphism of sites π described above. More precisely the

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