ILLINOIS JOURNAL OF MATHEMATICS
Volume 29, Number 4, Winter 1985

PRODUCT OF SUBGROUPS IN LIE GROUPS

Dedicated to Professor Gail S. Young on His Seventieth Birthday

BY

TA-SUN WU[1]

Introduction

In the study of analytic subgroups of an analytic group, sometimes it is important to know the structure of the product of two analytic subgroups and the closure of this product. There are two fundamental theorems in this aspect: one is a theorem of Mostow's and another is known as the Auslander-Wang-Zassenhaus theorem. Mostow's theorem (cf. [5]) says that if D is a closed uniform subgroup of a solvable analytic group G such that D contains no non-trivial normal analytic subgroup of G, then DN is closed in G and $D \cap N$ is a closed uniform subgroup of N, where N is the nilradical of G. (Let Y be a subset of a topological space X. Then we denote by $\overline{Y}$ the closure of Y in X. Let Z be a subgroup of a topological group K. Then, by definition, Z is a uniform subgroup of K or Z is uniform in K if $K/\overline{Z}$ is compact. If K is an analytic group, then the maximal nilpotent normal analytic subgroup of K is called the nilradical of K.) Clearly, Mostow's theorem holds for discrete uniform subgroups. A natural question is what happens if the discrete subgroup D is not uniform. In fact, in this case, it is easy to find examples of D so that DN is not closed in G and $D \cap N$ is not uniform in N. However, in these notes, we prove the following result.

THEOREM 1. *Let G be a simply connected solvable analytic group with nilradical N. If D is a discrete subgroup of G such that DN is dense in G, then D is nilpotent.*

By definition, if D is a discrete subgroup of a locally compact group G and if α: $G \to G/D$ denotes the canonical map, then D is called an L-subgroup of G provided that for every neighborhood U of the identity element e of G the

Received May 23, 1983.

[1]We would like to express our thanks to the referee for pointing out some errors in the earlier version of this article.