ILLINOIS JOURNAL OF MATHEMATICS
Volume 29, Number 4, Winter 1985

A NOTE ON FUCHSIAN GROUPS

BY

MORRIS NEWMAN[1]

Introduction

The origin of this note was in a query to the author from J. Lehner and M. Sheingorn, asking for a proof of Theorem 2 below. This theorem follows directly from Theorem 1 below, which is itself an easy consequence of known theorems, such as the Frobenius induced representation theorem, but does not seem to have been noticed previously. In the author's opinion this theorem is of genuine interest, since integral matrix groups may be treated arithmetically, and so may cast light on the more complicated fuchsian groups.

I am indebted to M. Tretkoff for supplying many references to the literature on theorems of the type of Theorem 2, and for numerous interesting remarks. In particular paper [6] by P. Scott contains a proof of Theorem 2 for surface groups, using hyperbolic geometry as the tool; and M. and C. Tretkoff have given a proof using covering spaces.

I am also indebted to W. Magnus for numerous valuable comments and suggestions. Most of the necessary background material for this note may be found in his book on the noneuclidean tessellations [4].

Finally, I am indebted to R. Lyndon for reading a preliminary version of this note and for suggesting a number of additions and improvements which have been incorporated into the text.

The theorem of Frobenius referred to above is as follows: Let G, H be groups such that $G \supset H$, $(G\colon H) = \mu < \infty$. Let α be a faithful representation of H of degree n. Then α induces a faithful representation β of G of degree μn, and β is integral if α is integral. A convenient reference for this theorem is Boerner's book [1].

Now suppose that G is a finitely generated fuchsian group. Then as an abstract group G is generated by elements

$$E_1, E_2, \ldots, E_s;\ P_1, P_2, \ldots, P_t;\ A_1, B_1, A_2, B_2, \ldots, A_g, B_g.$$

Any one of s, t, g may be 0, but to avoid degeneracy, it is assumed that

Received May 5, 1983.
[1]This work was supported by a grant from the National Science Foundation.