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FLAT FAMILIES OF AFFINE LINES ARE AFFINE-LINE BUNDLES

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By a *fibration* over a scheme S we mean a faithfully flat morphism $X \to S$ of finite type, and one such morphism will be called an A^n*-bundle* over S if the S-scheme X is locally isomorphic to the affine n-space A^n_s and S relative to the Zariski topology. The purpose of this note is to establish the following theorem.

THEOREM. *If $X \to S$ is a fibration such that for each point $s \in S$ the fiber of s in X is isomorphic to the affine line $A^1_{k(s)}$ and if the base S is a Noetherian normal scheme, then X is an A^1-bundle over S.*

Earlier, Kambayashi and Miyanishi proved this theorem under the additional hypotheses that the morphism $X \to S$ be affine and the base S be locally factorial [7; Theorem 1]. On the other hand, they assumed only the generic fiber to be an A^1 and all other fibers to be geometrically integral. The proof in the present paper reduces to the case where S is the spectrum of a local ring, then proceeds by induction on the dimension of S, which was the approach of [7]. The proof for $\dim S = 1$ when $X \to S$ is assumed to be affine was quite elementary (see [7; Lemma 1.3]); without this assumption it is much less so. The crux of our argument for the $\dim S = 1$ case (see §1) employs a lemma of Swan, which appears with proof as Lemma 1.1 below. The main task when $\dim S > 1$ is to weaken the assumption of "locally factorial" down to "normal" for the base scheme S. This is done in §2 following an idea of V. Danilov, contained in a letter to one of the authors. His idea involves a reduction to the case of a Henselian base scheme S, and a clever "two section" argument for that case. We are much indebted to him and want him to receive the appropriate credit for his vital contribution to this proof. We also thank I. Dolgachev for clarifying some of the points in Danilov's letter.

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