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THE CARDINALITY OF THE SET OF LEFT INVARIANT MEANS ON A LEFT AMENABLE SEMIGROUP

BY

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1. Introduction

The study of the cardinality of the set of invariant means on a (discrete) group essentially goes back to Banach ([1], [2]) who showed that there exist at least two invariant means on the circle group. In his famous paper [5] of 1957, Day continued the study, showing that many infinite amenable groups admit more than one left invariant mean. Following progress made by Granirer, C. Chou ([4]) obtained in 1976 the following definitive result: *the cardinality of the set* $\mathfrak{L}(G)$ *of left invariant means on an infinite amenable group* G *is*

$$2^{2^{|G|}} (= |l_\infty(G)'|!)$$

where $|E|$ is the cardinality of a set E. The method used by Chou has become canonical. The idea (expressed precisely in (3.1)) is to construct a "large" disjoint family $\mathscr{A}$ of subsets of G, each of which supports a left invariant mean, then to close up these subsets and their complements in the Stone-Cěch compactification βG, and then to find an even larger family of left invariant means supported on intersections of translates of these sets. This procedure is also followed in the present paper which deals with the result corresponding to Chou's for a left amenable semigroup S.

The semigroup case is substantially more difficult than the group case owing to former's more complicated multiplication structure.

Luthar [16] obtained the first positive result for the semigroup case: *if* S *is abelian, then* S *has more than one invariant mean if and only if it does not contain a finite ideal.* Granier [8]–[10] showed, along with other results, that $|\mathfrak{L}(S)|$ is infinite if S is infinite, left amenable and left cancellative. Chou [3] showed that if S is infinite and cancellative, then $|\mathfrak{L}(S)| \geq 2^{\aleph_0|S|}$. The work of Granirer [8], [9] and Klawe [14] led to the following definitive result dealing with the case where the span $\mathfrak{I}_l(S)$ of the set of left invariant means on S is

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