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HECKE MODULAR FORMS AND q-HERMITE POLYNOMIALS

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1. Introduction

In this paper we shall use a technique of L.J. Rogers, expansion in terms of q-Hermite polynomials,

$$A_n(\cos\theta|q) = \sum_{i=0}^{n} \begin{bmatrix} n \\ i \end{bmatrix} \cos(n-2i)\theta, \tag{1.1}$$

where

$$\begin{bmatrix} n \\ i \end{bmatrix} = \prod_{j=1}^{i} \frac{(1-q^{n-i+j})}{(1-q^j)}$$

is the Gaussian polynomial, to derive a number of identities which express a summation of the form

$$\sum_{(n,m)\in D} (-1)^{f(n,m)} q^{Q(n,m)+L(n,m)} \tag{1.2}$$

as a rational product of η-functions, where Q is a quadratic form, L is a linear form and $D \subseteq \{(n,m) \in \mathbf{Z}\times\mathbf{Z} | Q(n,m) \geq 0\}$.

The most famous identity of this type is due to Jacobi [7, Theorem 357]:

$$\prod_{n\geq 1}(1-q^n)^3 = \sum_{m=-\infty}^{\infty} \sum_{n\geq|m|} (-1)^n q^{(n^2+n)/2} \tag{1.3}$$

$$= \sum_{n\geq 0} (-1)^n (2n+1) q^{(n^2+n)/2}.$$

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