ILLINOIS JOURNAL OF MATHEMATICS
Volume 30, Number 2, Summer 1986

ON DEHN PRESENTATIONS AND DEHN ALGORITHMS

BY

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Dedicated to the memory of William Werner Boone

The origin of this note is the observation that somewhat stronger results can be obtained from certain proofs of theorems yielding the applicability of Dehn's algorithm for the word problem to one or another class of groups. This has undoubtedly been noticed by many, but never stated, because the stronger results have less elegant formulations and no known additional applications. However, some of their analogues for groups in larger classes do have additional applications, allowing simplifications of proofs and algorithms. The reader is assumed to be familiar with Chapter V of [4]. The smallest number of pieces into which a word w can be decomposed will be denoted by $\|w\|$. Pieces will mean non-empty pieces. Diagrams of minimal R-sequences will be called minimal R-diagrams. Words will mean cyclic words whenever possible.

We now define various classes of Dehn presentations. Let $G = \langle \mathbf{X}; R \rangle$, where R is symmetrized.

DEFINITION 1. G is a weak, strict, metric Dehn presentation iff given any freely reduced word $w = 1$ in G, there exist a subword s of w and a word t such that $s\bar{t} \in R$ and $|s| > |t|$.

DEFINITION 2. G is a strong, strict, metric Dehn presentation iff given any freely reduced word $w = 1$ in G and any minimal R-diagram M for w, there exist boundary region D and words s, t such that $s\bar{t}$ is a label of ∂D, s is the label of $\partial D \cap \partial M$, a consecutive part of M, and $|s| > |t|$.

Definitions 3 and 4 are obtained from Definitions 1 and 2, respectively, by omitting "strict" and replacing " $>$ " by " $\geq$ ".

Definitions 5, 6, 7 and 8 are obtained from Definitions 1, 2, 3 and 4, respectively, by changing "metric" to "non-metric" and "$|\ \ |$" to "$\|\ \ \|$".

Conjecture. The finitely presented subclass of each of the above classes is non-recursive.

Received December 2, 1985.