ILLINOIS JOURNAL OF MATHEMATICS
Volume 30, Number 2, Summer 1986

ON THE NOVIKOV AND BOONE-BORISOV GROUPS

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In Memoriam W.W. Boone

1. Introduction

In the history of word problems in group theory the fundamental role was played by pioneering works of P.S. Novikov [1] and W. Boone [2]. The construction by Novikov in [1] of the centrally-symmetric group $\mathfrak{A} = \mathfrak{A}_{pd\mu l\rho}$ has never been given any further analysis different from [1]. The construction of Boone's group $G(T, q)$ [2] was analysed by many authors who introduced a number of groups which may be called the modifications of Boone's construction (for example, see [3], [4], [5]). One of these modifications is the construction due to V.V. Borisov [6]. We call the group $\Gamma(\Pi, P)$ from Borisov's work the Boone-Borisov group.

Our aim in this note is to make a survey of the author's recent results on the groups $\mathfrak{A}$ and $\Gamma(\Pi, P)$. The group $\mathfrak{A}$ has the "big" subgroup $\mathfrak{A}_{d\mu l\rho}$.

THEOREM 1. *Novikov's group* $\mathfrak{A}_{d\mu l\rho}$ *has a standard basis.*

This theorem was announced by the author in [7]. Theorem 1 provides a comparatively short proof for the criterion of equality of words in $\mathfrak{A}_{d\mu l\rho}$ which is the main theorem of chapters I-IV of [1] (the remaining two chapters V, VI of [1] treat some nongroup combinatorial calculus).

THEOREM 2. *The Boone-Borisov group* $\Gamma(\Pi, P)$ *has a standard basis.*

From Theorem 2 it is comparatively easy to deduce that the word problem in $\Gamma(\Pi, P)$ is Turing (or even truth-table) equivalent to the problem of the equality to the word P in the initial semigroup Π. Since for any Turing (truth-table) degree of unsolvability α there exists a f.p. semigroup in which for example the problem of the equality to the empty word has just the given degree of unsolvability, it follows that the Boone-Borisov group may have arbitrary Turing (truth-table) degree of unsolvability. The existence of f.p.

Received Sept. 10, 1985.