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SOME QUESTIONS OF EDJVET AND PRIDE ABOUT INFINITE GROUPS

BY

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Dedicated to the memory of Bill Boone

1. The height of Pride

In his paper [9] Stephen Pride describes a pre-order $\preccurlyeq$ on the class of groups. In effect, as modified slightly in [2] the definition is that $H \preccurlyeq G$ if there exist:

$$(*)\qquad \begin{cases} \text{(i)}\quad \text{a subgroup } G_0 \text{ of finite index in } G \\ \text{and a normal subgroup } G_1 \text{ of } G_0; \\ \text{(ii)}\quad \text{a subgroup } H_0 \text{ of finite index in } H \\ \text{and a finite normal subgroup } H_1 \text{ of } H_0; \\ \text{(iii)}\quad \text{an isomorphism } G_0/G_1 \to H_0/H_1. \end{cases}$$

If $H \preccurlyeq G$ and $G \preccurlyeq H$ then we write $G \sim H$, and we use $[G]$ to denote the equivalence class consisting of all such groups H. The relation $\preccurlyeq$ induces a partial order, also denoted $\preccurlyeq$, on the collection of all equivalence classes, with the class $[\{1\}]$ of all finite groups as its unique least member. The ideal $\mathrm{Id}[G]$ is defined to be the partially ordered set consisting of all equivalence classes $[H] \preccurlyeq [G]$. A group G is said to be *atomic* if $\mathrm{Id}[G]$ consists of $[\{1\}]$ and $[G]$; it is said to be of *height* h, and we write $\mathrm{ht}[G] = h$, if $\mathrm{Id}[G]$ is of height h as partially ordered set. In the papers [2], [9] a number of questions about these concepts are raised. These, and one or two others, are stated in §2 below. Answers are given in §§3–8. In a final section (§9) I prove some small results relating the pre-order $\preccurlyeq$ and the property max-N. The fact that a finitely generated atomic group satisfies max-N is typical of these.

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