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DIAGONAL EMBEDDINGS OF NILPOTENT GROUPS

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Dedicated to the memory of William Boone

1. Introduction

Among various embeddings of a group G into $G \times G \times G$ are the embeddings

$$\phi_1: g \to (g, g, 1) \quad \text{and} \quad \phi_2: g \to (1, g, g)$$

which yield a weak form of permutability between the isomorphic groups G^{ϕ_1} and G^{ϕ_2}, namely, $g^{\phi_1}g^{\phi_2} = g^{\phi_2}g^{\phi_1}$ for all $g \in G$. This natural situation leads to the study of the double group

$$\mathbf{D}(G) = \langle G^{\phi_1}, G^{\phi_2};\ g^{\phi_1}g^{\phi_2} = g^{\phi_2}g^{\phi_1} \quad \text{for all } g \in G\rangle$$

as the quotient group of the free product $G^{\phi_1} * G^{\phi_2}$ by the commutator relations $[g^{\phi_1}, g^{\phi_2}] = 1$ for all $g \in G$. When G is finite, $\mathbf{D}(G)$ is finite (Sidki [4]), and when G is a finite p-group of order p^k, p odd, $\mathbf{D}(G)$ is of order dividing $p^{2k}p^{k(k-1)/2}$ (Rocco [3]). In this paper we develop commutator calculus for the double group $\mathbf{D}(G)$ and obtain a detailed description of its lower central series $\gamma_i(\mathbf{D}(G))$, $i \geq 1$, in terms of the lower central series of G. We prove that *if G is an m-generator nilpotent group of class at most c with $m \geq 2$, $c \geq 1$, then $\mathbf{D}(G)$ is nilpotent of class at most* $\max\{m, c+2\}$. *Furthermore, if $m \geq c+3$ then $\gamma_{c+3}(\mathbf{D}(G))$ is an elementary abelian 2-group of rank at most*

$$\sum_{k=c+3}^{m} \binom{m}{k}$$

(Theorems 3.2 and 3.3).

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