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RECOGNIZING FREE METABELIAN GROUPS

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Dedicated to the memory of W.W. Boone

1. Introduction

For groups in general many algorithmic problems are known to be recursively unsolvable. But for some special classes of groups one can give algorithms for solving certain decision problems—for example, there is a well-known algorithm to solve the isomorphism problem for finitely presented abelian groups. This paper is concerned with finitely generated metabelian groups. The isomorphism problem for this class is as yet unresolved, but we show there is an algorithm to determine whether or not a suitably given finitely generated metabelian group is free metabelian. A useful algebraic characterization of free metabelian groups is also obtained.

2. Some presentations, algorithms and observations

Let G be a finitely generated metabelian group and $A = [G, G] = G'$ its commutator subgroup. Because finitely generated metabelian groups satisfy max-n, the maximum condition for normal subgroups, G can be defined by finitely many generators subject to the relations which are consequences of the metabelian law plus finitely many additional relations. So G can be presented in the form

$$(1) \qquad G = \langle x_1, \ldots, x_n;\ r_1 = 1, \ldots, r_m = 1,\ G'' = 1 \rangle$$

where the r_j's are certain words in the x_i and $G'' = 1$ represents the infinitely many relations corresponding to the metabelian law. We call this a *finite metabelian presentation of* G. Of course such a presentation is a finite description of G even though G may not be finitely presented in the usual sense.

Again because G satisfies max-n, its commutator subgroup $A = [G, G]$ is a finitely generated $\mathbf{Z}G$-module where G acts on A by conjugation. Putting

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