

DECISION PROBLEMS FOR SOLUBLE GROUPS OF FINITE RANK

BY

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In Memoriam W.W. Boone

1. Introduction and results

In the present work we shall be concerned with three classical decision problems of group theory, the word problem, the generalized word problem and the conjugacy problem. It is now known that all three problems have negative solutions even in the class of finitely presented soluble groups. The first example of a finitely presented soluble group with insoluble word problem was given by Harlampovič [8]. Further examples have been found by Baumslag, Gildenhuys and Strebel [3].

In the light of these negative results it is of interest to discover finiteness conditions which are strong enough to imply solubility of one or more of the three decision problems. Indeed some such conditions are already known. Baumslag, Cannonito and Miller [1] showed that the word problem is soluble for nilpotent-by-polycyclic-by-finite groups which satisfy max-n, the maximal condition on normal subgroups; in particular this conclusion applies to finitely generated abelian-by-polycyclic-by-finite groups, by a well-known theorem of P. Hall [7]. At this point it is as well to note that Harlampovič's example has derived length 3 and is nilpotent of class 4-by-abelian; of course the group does not have max-n. Another cautionary remark; there are soluble groups of derived length 3 satisfying max-n which are not recursively presentable and thus have insoluble word problem; such examples are constructed in [16]. However it remains an open question whether a finitely presented (or even recursively presented) soluble group with max-n necessarily has soluble word problem.

Another positive result on the word problem was recently obtained by Cannonito and Robinson [4]; it was shown that a finitely generated soluble group with finite Prüfer rank has soluble word problem if and only if the

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