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THE VARIATION OF VECTOR MEASURES AND CYLINDRICAL CONCENTRATION

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The mutual relationship between the differentiability of vector measures and the smoothness properties of the associated cylindrical measures has often been used in the study of measure theory on vector spaces; see for example A. Goldman [4]. In [3] it was shown that a lifting is not always the best method for constructing weak densities for vector measures. If the notion of a "density" is relaxed somewhat, then it follows that a vector measure m will be differentiable with respect to a probability λ when the associated (m, λ)-distribution is σ-additive. Conversely, the existence of this weaker type of density suffices to guarantee the σ-additivity of the (m, λ)-distribution. We give a few examples to show the limitations of these techniques.

The construction of a regular density for a vector measure usually involves an argument utilizing its average range with respect to a probability. By appropriately defining the notion of the variation of a vector measure, we point out that the differentiability of a vector measure is an intrinsic property, independent of the associated scalar measure—a simple observation that allows us to complete the cycle relating the variation of m to the average range of m with respect to λ, and the cylindrical concentration of the (m, λ)-distribution.

In a number of locally convex spaces, indefinite integrals have special variational properties. By using the new notion of the variation of a vector measure, we give a number of conditions ensuring that a locally convex space has a form of variation property which has a bearing on the regularity of cylindrical measures defined on the space. As a by-product, a necessary and sufficient condition for the existence of a density for a vector measure with values in a space of regular Borel measures is given; it does not seem to have been stated explicitly previously.

Let E be a locally convex space. Let $\mathscr{Z}(E)$ be the smallest algebra, and $\mathscr{C}(E)$ the smallest σ-algebra for which elements of E' are measurable. The families $\mathscr{Z}(E)$ and $\mathscr{C}(E)$ contain all sets of the form $\varphi^{-1}(B)$ where $\varphi: E \to \mathbf{R}^k$ is a continuous linear map and B is a Borel subset of $\mathbf{R}^k$.

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