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PERIODICITY IN THE COHOMOLOGY OF UNIVERSAL G-SPACES

BY

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INTRODUCTION

The purpose of this note is to generalize the classical results on periodicity in $H^*(G)$ in the presence of a free G-action on a sphere, and to reinterpret them in terms of global results about equivariant singular cohomology.

Our generalizations proceed in two directions. First, one has a notion of $H^*(G; T)$, where T is a Mackey functor (in the sense of tom Dieck in [2]), generalizing the case T a $\mathbf{Z}G$-module. We show here that classical periodicity continues to hold in this more general setting.

Next, one has the notion of a universal G-space $E\mathscr{F}$, associated with a family $\mathscr{F}$ of subgroups of G. Here, we exhibit periodicity in $H_G^*(E\mathscr{F}; T)$ (for arbitrary G and particular families $\mathscr{F}$), where * is $RO(G)$-grading. (The theory of $RO(G)$-graded equivariant singular cohomology has been announced by Lewis, May, and McClure in [4]. The complete theory will appear in [5], including one of the author's independent formulations, a summary of which appears in §1 below). This periodicity is seen to arise from a "Bott" class $1_V \in H_G^V(\text{point})$ for appropriate representations V, in the sense that $\cup 1_V$ is an isomorphism in a range. Further, we see that this class lies at the source of the classical periodicity results, which emerge as special cases.

Finally, we use the periodicity to extend the computation of $H_G^n(E\mathscr{F}; T)$ carried out in [7] and [8] to that of $H_G^{nV+m}(E\mathscr{F}; T)$ for $m, n \geq 0$ and $\mathscr{F}$ a family of subgroups determined by V. These latter groups (which are also modules over the Burnside ring of G) turn out to be purely algebraic invariants of G and V. (Throughout, G will be a finite group.)

1. Equivariant RO(G)-graded singular cohomology

We recall here in brief some of the theory of equivariant $RO(G)$-graded singular cohomology, developed by Lewis, May, McClure and the author in [5].

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