ILLINOIS JOURNAL OF MATHEMATICS
Volume 30, Number 3, Fall 1986

A BMO ESTIMATE FOR MULTILINEAR SINGULAR INTEGRALS

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Introduction

Let

$$(1)\quad C(a,b;f)(x) = \text{p.v.}\int_{\mathbf{R}^n} \frac{\Omega(x-y)P_{m_1}(a;x,y)P_{m_2}(b;x,y)f(y)\,dy}{|x-y|^{n+M-2}}$$

where

$$P_m(a,x,y) = a(x) - \sum_{|\alpha|<m} \frac{1}{\alpha!}a^{(\alpha)}(y)(x-y)^\alpha$$

and $M = m_1 + m_2$. In this paper we establish the inequality

$$(2)\quad \|C(a,b;f)\|_p \le C_p\|\nabla^{m_1-1}a\|_{BMO}\|\nabla^{m_2-1}b\|_{BMO}\|f\|_p,\quad 1<p<\infty,$$

where Ω satisfies certain conditions and $\|\nabla^m a\|_{BMO} = \sum_{|\alpha|=m}\|a^{(\alpha)}\|_{BMO}$. BMO denotes the space of functions of bounded mean oscillation on $\mathbf{R}^n$.

The first result in this direction was established by Coifman, Rochberg, and Weiss [7] where it was shown that the commutator of the Hilbert transform and multiplication by a function A is bounded on $L^p(\mathbf{R})$, $1<p<\infty$, providing A is in BMO. The result for a single remainder of order 2 was proved by the first author in [3]. The methods used here are extensions of those in [3]. The main differences are: (1) a generalization of a basic estimate of Mary Weiss to Taylor series remainders (our lemma); (2) the boundedness of operators similar to $C(a,b;f)$ when a and b have appropriate derivatives in $L^q(\mathbf{R}^n)$ (see [2]); (3) a more complicated partition of the operator due to the presence of products and the fact that the order of the remainders is arbitrary.

Finally we note that the result proved here holds for any finite number of remainders. For simplicity we give the proof here for the case of two remainders. The authors wish to point out that while going from one re-

Received September 28, 1983.