ILLINOIS JOURNAL OF MATHEMATICS
Volume 30, Number 3, Fall 1986

MEROMORPHIC AND RATIONAL FACTORS OF AUTOMORPHY

BY

DAVID A. JAMES

I. Introduction

Henri Cartan [2] illustrated the appeal of factors of automorphy as a general approach to automorphic forms, and Gunning [12] gave other applications of these factors. Several contributions to the area of factors of automorphy were recently summarized in [14], but to these must be added the work of Rankin [17, p. 70 ff.] and of Christian [3], [4], [5], [6] in the Siegel upper half plane of degree $n > 1$, and of Gunning [11].

A factor of automorphy $v(z, \phi)$ on $D \times \Gamma$ satisfies the consistency condition

$$v(z, \phi \circ \Psi) = v(z, \Psi) v(\Psi z, \phi)$$

for all ϕ and Ψ in a group Γ of homeomorphisms of D onto itself. In this paper we consider the specific case in which D is the complex plane and $\Gamma \subset SL(2, R)$. For each M in Γ there is associated the homeomorphism

$$Mz = (az + b)/(cz + d).$$

The consistency condition becomes

$$v(z, MN) = v(z, N) v(Nz, M) \quad \text{for all } M, N \text{ in } \Gamma.$$

Two familiar factors of automorphy are v identically equal to one, and

$$v(z, M) = u(M)(cz + d)^k \quad \text{for all } M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ in } \Gamma.$$

Given a factor of automorphy v, there is customarily an associated function $f(z)$ with the property that $f(Mz) = v(z, M) f(z)$ for all M in Γ. In the first case, $f(Mz) = f(z)$ so f is an (unrestricted) automorphic function, and, in the

Received April 15, 1983.