ILLINOIS JOURNAL OF MATHEMATICS
Volume 31, Number 1, Spring 1987

SURFACES WITH MINIMAL RANDOM WEIGHTS AND MAXIMAL FLOWS: A HIGHER DIMENSIONAL VERSION OF FIRST-PASSAGE PERCOLATION

BY

HARRY KESTEN[1]

1. Introduction

Random surfaces with independent plaquettes were studied by Aizenman et al. [2] in connection with bond percolation in $\mathbf{Z}^3$. Such surfaces arise naturally as dual objects to percolation clusters, and they exhibit critical phenomena similar to other percolation models (cf. [2]). Random surfaces with dependent plaquettes have recently been considered in several other physical contexts [1], [6] but here we stick to situations where the plaquettes have independent characteristics. Our original motivation was to generalize the estimates on the resistances of random two-dimensional subnetworks of $\mathbf{Z}^2$ of [8] and [12, Ch. 11], to three-dimensional subnetworks of $\mathbf{Z}^3$. Since our results in this direction are still unsatisfactory (as indicated in (2.23) below) we concentrate here on the relation with first-passage percolation and maximal flows.

We begin with a brief description of the fundamental results of first-passage percolation on $\mathbf{Z}^d$ (d not necessarily restricted to 2 or 3). A good introduction to the subject is the monograph [15] of Smythe and Wierman. For later results see also [13]. To each edge e of $\mathbf{Z}^d$ between two neighboring vertices[2] of $\mathbf{Z}^d$ one assigns a random nonnegative value $t(e)$. It is assumed that all $t(e)$, $e \in \mathbf{Z}^d$, are independent and have the same distribution function F with

$$(1.1) \qquad F(0-) = 0;$$

$t(e)$ was interpreted by Hammersley and Welsh in [10]—the article which started the subject—as the passage time of e. A *path* on $\mathbf{Z}^d$ (from v_0 to v_n) is a sequence $(v_0, e_1, v_1, \ldots, e_n, v_n)$ of vertices $v_0, \ldots, v_n$ alternating with edges $e_1, \ldots, e_n$, such that v_{i-1} and v_i are neighbors on $\mathbf{Z}^d$ with e_i the edge of $\mathbf{Z}^d$

Received February 15, 1985.

[1]Research supported by the National Science Foundation through a grant to Cornell University.

[2]The vertices $v = (v(1), \ldots, v(d))$ and w in $\mathbf{Z}^d$ are neighbors (or adjacent) if and only if $\Sigma_1^d |v(i) - w(i)| = 1$.