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TWO SPACE SCATTERING AND PROPAGATIVE SYSTEMS

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1. Introduction

Wilcox [27] showed that many wave propagation phenomena of classical physics are governed by systems of partial differential equations of the form

$$E(x)\frac{\partial u}{\partial t} = \sum_{j=1}^{n} A_j \frac{\partial u}{\partial x_j} \equiv -iAu \tag{1.1}$$

where $x = (x_1, \dots, x_n) \in \mathbf{R}^n$, $u(x, t)$ is a column vector of length m describing the state of the medium at position x and time t, and $E(x)$ and the A_j are $m \times m$ matrices with the following properties:

(a) $E(x)$ is real, symmetric and uniformly positive definite.

(b) The A_j are real, symmetric and constant.

From the point of view of spectral and scattering theory it is desirable that the solution of (1.1) be of the form

$$u = e^{-itH}u_0, \quad u_0(x) = u(x,0) \tag{1.2}$$

where H is a self adjoint operator. This would require that H be an extension of $E^{-1}A$. When $E = 1$, one can easily obtain a self adjoint realization H_0 of A in $\mathscr{H} = (L^2)^m$ using Fourier transforms. On the other hand, if $E \neq 1$, the operator $E^{-1}A$ need not be Hermitian on $\mathscr{H}$. However, it is Hermitian on the Hilbert space $\mathscr{H}_1$ with scalar product

$$(u, v)_1 = \int v(x)^* E(x) u(x)\, dx. \tag{1.3}$$

If $E(x)$ is uniformly bounded, it can be shown that $E^{-1}H_0$ is self adjoint on $\mathscr{H}_1$ (cf. [27]). However, when $E(x)$ is unbounded, it need not be self adjoint.

In the present paper we give sufficient conditions on the matrix $E(x)$ for the operator $E^{-1}A$ to have a self adjoint extension H on $\mathscr{H}_1$. We then study the

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