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MODULES OF FINITE LENGTH AND CHOW GROUPS OF SURFACES WITH RATIONAL DOUBLE POINTS

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Let R be a local ring, and let $\mathscr{C}_R$ denote the category of R-modules of finite length and finite projective dimension. The Grothendieck group $K_0(\mathscr{C}_R)$ is defined, as usual, to be the quotient $\mathscr{F}/\mathscr{R}$ where $\mathscr{F}$ is the free abelian group on isomorphism classes of objects of $\mathscr{C}_R$, and $\mathscr{R}$ is the subgroup of $\mathscr{F}$ generated by elements $[M] - [M'] - [M'']$ corresponding to exact sequences

$$0 \to M' \to M \to M'' \to 0$$

in $\mathscr{C}_R$. Note that $\mathscr{C}_R$, and hence $K_0(\mathscr{C}_R)$, depends only on the analytic isomorphism class (i.e., the completion $\hat{R}$) of R, since modules of finite length are complete. Also, if R is regular, $K_0(\mathscr{C}_R)$ is just $\mathbf{Z}$, since in $K_0(\mathscr{C}_R)$ a module of length l is equivalent to l copies of the residue field.

If $\dim R = 2$, we say that R has a *rational double point* if the completion $\hat{R}$ is isomorphic to $\hat{\mathscr{O}}_{P,X}$ where $P \in X$ is the local ring of a rational double point P on a surface X over an algebraically closed field k. Thus if k has characteristic 0, $\hat{R}$ is isomorphic to $k[[x, y, z]]/(f(x, y, z))$ where f is one of the following:

$$\begin{array}{ll} z^{n+1} + xy & (A_n) \\ z^2 + xy^2 + x^{n+1} & (D_{n+2}),\ n \geq 2 \\ z^2 + y^3 + x^4 & (E_6) \\ z^2 + x^3y + y^3 & (E_7) \\ z^2 + x^3 + y^5 & (E_8). \end{array}$$

We can now state our main result.

THEOREM 1. *Let R be a 2-dimensional (noetherian) local ring with algebraically closed residue field k of characteristic 0. Suppose that R has a rational double point. Then $K_0(\mathscr{C}_R) = \mathbf{Z}$.*

We have the following geometric consequence. For a normal quasi-projective surface X, the Chow group of zero cycles $F_0K_0(X)$ is defined to be the

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