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TENSOR PRODUCTS OF TSIRELSON'S SPACE

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Tsirelson's space T has attracted considerable interest during the past few years, somewhat eclipsing the original space T^* discovered in 1973 by B. S. Tsirelson [12]. However, in [1], the first two authors and Dineen showed that T^* held the greater interest, from the point of view of holomorphic functions. Specifically, the main result of [1] is that for all positive integers n, $P({}^nT^*)$ is reflexive. As a consequence, it is shown that the space $(H(T^*), \tau_\omega)$ of complex-valued holomorphic functions on T^*, endowed with the Nachbin ported topology, is reflexive. Here, we continue our study of multilinear properties of T^* by showing that $P({}^nT^*)$ is "Tsirelson-like", in the sense that it is reflexive, with (not unconditional) basis, and contains no l_p space for $1 < p < \infty$. In fact, our method of proof enables us to prove that $(H(T^*, l_p), \tau_\omega)$ and $P({}^nT^*, l_p)$ are reflexive for all $n = 1, 2, \ldots$ and all p, $1 < p < \infty$.

Our notation and terminology will follow the earlier paper [1]. Given Banach spaces X and Y, $L({}^nX, Y)$ is the Banach space of continuous n-linear mappings $A\colon X \times \cdots \times X \to Y$, with norm

$$\|A\| = \sup\left\{ \|A(x_1, \ldots, x_n)\| \colon x_j \in X,\ \|x_j\| \le 1, 1 \le j \le n \right\}.$$

$L({}^nX)$ denotes $L({}^nX, K)$ where $K = R$ or C. An important observation for us will be the fact that $L({}^nX, Y)$ is isometrically isomorphic to the space $L(\hat{\otimes}_\pi^n X, Y)$ of linear mappings between the n-fold completed projective tensor product of X with itself and Y. Similarly the space $L_s({}^nX, Y)$ of symmetric continuous n-linear mappings is isometrically isomorphic to the space $L(ⓢ^n X, Y)$, where $ⓢ^n X$ is the symmetric n-fold completed projective tensor product of X with itself. $L_s({}^nX, Y)$ is also isomorphic to the Banach space $P({}^nX, Y)$ of n-homogeneous continuous polynomials from X to Y, where each element $P \in P({}^nX, Y)$ is defined as $P(x) = A(x, \ldots, x)$ for a unique element $A \in L_s({}^nX, Y)$. For basic properties of tensor products, we refer to [3] (See also [11]). See [4] for any unexplained notation and definitions from infinite dimensional holomorphy.

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