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THE DISTANCE TO THE ANALYTIC TOEPLITZ OPERATORS

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The algebra $\mathcal{T}(H^\infty)$ of all analytic Toeplitz operators is a reflexive, maximal abelian subalgebra of $\mathcal{B}(H^2)$. Thus there are three natural measures of how far an operator A is from $\mathcal{T}(H^\infty)$, namely

$$d(A) = \inf_{h \in H^\infty} \|A - T_h\|,$$
$$\delta(A) = \sup_{h \in H^\infty, \|h\|_\infty = 1} \|AT_h - T_h A\|$$

and

$$\beta(A) = \sup_{\omega \text{ inner}} \|P_\omega^\perp A P_\omega\|$$

where $P_\omega = T_\omega T_\omega^*$ is the orthogonal projection onto any invariant subspace of $\mathcal{T}(H^\infty)$. It is immediate that all three of these measures vanishes precisely on $\mathcal{T}(H^\infty)$. It will be shown that they are comparable. More precisely:

THEOREM 1. *Let A belong to $\mathcal{B}(H^2)$. If A is lower triangular,*

$$\beta(A) \le d(A) \le \delta(A) \le 2d(A) \le 18\beta(A).$$

In general,

$$\tfrac{1}{2}d(A) \le \delta(A) \le 2d(A) \quad \textit{and} \quad \beta(A) \le d(A) \le 19\beta(A).$$

THEOREM 2. *Let $\mathcal{T}$ be a unital, weak* closed subalgebra of Toeplitz operators. Then for any A in $\mathcal{B}(H^2)$,*

$$d(A, \mathcal{T}) \le 39 \sup\{\|P^\perp AP\| : P \in \operatorname{Lat} \mathcal{T}\}.$$

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