ILLINOIS JOURNAL OF MATHEMATICS
Volume 32, Number 1, Spring 1988

CONVERGENCE RATES FOR FUNCTION CLASSES WITH APPLICATIONS TO THE EMPIRICAL CHARACTERISTIC FUNCTION

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1. Introduction

Let $(S, \mathscr{S}, P)$ be a probability space and let X_i, $i \geq 1$, be independent, identically distributed (i.i.d.) S-valued random variables with common law P. We shall consider the X_i, $i \geq 1$, to be the coordinates for a countable product $(S^{\mathrm{N}}, \mathscr{S}^{\mathrm{N}}, P^{\mathrm{N}})$ of copies of $(S, \mathscr{S}, P)$. Let the nth empirical measure for P be defined by

$$P_n := n^{-1}(\delta_{X_1} + \cdots + \delta_{X_n}),$$

where δ_x is the unit mass at $x \in S$.

Recent research has yielded new limit theorems for the empirical process

$$\left\{\int f(dP_n - dP) : f \in \mathscr{F}\right\},$$

where $\mathscr{F}$ is a class of measurable functions on S. We refer the reader to [5], [7], [10], [11], [22], [25] where attention is focused on the empirical process indexed by a single class of functions $\mathscr{F}$. Related research has concentrated on the empirical process indexed by a sequence of classes of functions, say $\mathscr{G}_n$, $n \geq 1$. For example, see [14], [21], [28], [31].

In recent work [31], the author has used randomization techniques and metric entropy methods to study the limit behavior of

$$\left\{\int g(dP_n - dP) : g \in \mathscr{G}_n\right\} \tag{1.1}$$

where $\mathscr{G}_n$, $n \geq 1$, is a sequence of function classes on $(S, \mathscr{S}, P)$. Under weak metric entropy and growth conditions on $\mathscr{G}_n$, $n \geq 1$, it is shown in [31], that

Received July 18, 1986.