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MIXING ACTIONS OF GROUPS[1]

BY

VITALY BERGELSON AND JOSEPH ROSENBLATT

0. Given a measure-preserving transformation T of a probability space (X,β, m), T is weakly mixing if and only if for all F_1, $F_2 \in L_2(X, \beta, m)$,

$$\lim_{N\to\infty} (1/N) \sum_{n=1}^{N} \left| \int F_1 T^n F_2 \, dm - \int F_1 \, dm \int F_2 \, dm \right| = 0. \tag{1}$$

This special extreme is important for recurrence theorems and can be characterized in a number of interesting ways, see Furstenberg, Katznelson, and Ornstein [8]. This concept was extended to amenable groups by Dye [5] and is closely related to properties of the unitary representations of G, see Schmidt [16]. By using the invariant mean on the weakly almost periodic functions, weakly mixing unitary representations on a Hilbert space H can be defined for all groups in a manner that directly extends (1). Moreover, by using the standard methods of harmonic analysis in von Neumann [13] and Godement [9], all the characterizations of weakly mixing actions hold here. This gives new and different proofs of these theorems in the cases studied in [5], [8], [16].

In Section 1, the general definition is discussed and many alternative characterizing properties of weakly mixing unitary representations are given. In Section 2, a category result shows that on the unitary level weakly mixing actions are residual for amenable groups. In Section 3, examples of special groups and properties of their actions due to the representations of the group are discussed. In Section 4, the previous abstract theory is summarized for actions induced on $L_2(X)$ by groups of measure-preserving transformations on X.

1. Let G be a σ-compact locally compact Hausdorff group. Such a group will be called a locally compact group. Let $\lambda = \lambda_G$ be a fixed left-invariant Haar measure on G. Let H be a separable infinite-dimensional Hilbert space and let τ be a continuous unitary representation of G on H. Let $CB(G)$ denote the continuous bounded functions on G, let $WAP(G)$ denote the weakly almost periodic functions on G, let $B(G)$ denote the Fourier-Stieltjes

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