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HANKEL OPERATORS IN VON-NEUMANN-SCHATTEN CLASSES

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Introduction

In [3], Bonsall reduced the study of a Hankel operator on the Hardy space H^2 of the disc D to the study of its action on a class of simple elements in H^2 which generate the space. To this end he introduced the unit vectors

$$v_z(w) = \frac{\sqrt{1-|z|^2}}{1-\bar{z}w} \quad (w \in D)$$

indexed by the points $z \in D$ and proceeded to show that A is a bounded Hankel operator if and only if $\{\|Av_z\|: z \in D\}$ is bounded. His methods also show that A is compact if and only if $\|Av_z\| \to 0$ uniformly as $|z| \to 1$.

The purpose of this paper is to try to find conditions which relate the quantities $\|Av_z\|$ with the property that A belongs to the von Neumann-Schatten class $\mathscr{C}_p (1 \leq p < \infty)$. We get a complete characterization only when $p = 2$. For other values of p we obtain implications in one direction only but are able to show that the converse implications do not hold.

Bonsall also considered unit vectors $u_n(\zeta)$, $n \geq 0$, $\zeta \in \partial D$, the counterparts of the v_z on the unit circle. We obtain completely analogous conditions in terms of $\|Au_n(\zeta)\|$ as stated above for $\|Av_z\|$ including a necessary condition that $A \in \mathscr{C}_1$. Again the condition is shown to be not sufficient.

Preliminaries

We record some notation we will use and recall some pertinent results. Let D denote the unit disc, ∂D the unit circle, $L^p = L^p(\partial D)$ the usual Lebesgue space, $0 < p \leq \infty$, and H^p, Hardy space, the subspace of L^p of functions analytic in D.

Let $\hat{f}(n)$ be the nth Fourier coefficient of the function f in L^1. We will follow the usual practice of identifying a function f in H^p with its analytic extension to D, $\sum_{n=0}^{\infty} \hat{f}(n) z^n$.

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