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TORSION FREE CANCELLATION OVER ORDERS

BY

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In memory of Irving Reiner

Let Λ be an order over a Dedekind ring R. We say that torsion free cancellation (hereafter abbreviated to TFC) holds for Λ if $X \oplus M \approx X \oplus N$ implies $M \approx N$ for lattices X, M, N over Λ; i.e. when X, M, N are finitely generated Λ modules torsion free over R. In [30], Wiegand developed a theory of torsion free cancellation over 1-dimensional commutative rings. Since the question is also of great interest for non-commutative orders, it is natural to ask whether Wiegand's results have non-commutative analogs. I will show here that this is indeed the case, at least when the quotient field of R is a global field. The main difference between the commutative and non-commutative case is due to the need to impose Eichler's condition on appropriate endomorphism rings.

I will also present some partial results on the case $\Lambda = ZG$ with G a finite group. The abelian case was discussed by Wiegand [30] who settled the question except for two special groups, the cyclic groups G_8 and C_9 of orders 8 and 9. It turns out that TFC holds also in these two cases.[1] For C_9 this can be deduced from Reiner's classification of ZC_{p^2} lattices [22]. The case C_8 requires a bit more work and will be discussed in §5. The final result for G abelian is that TFC holds for ZG if and only if $D(ZG) = 0$. However, I will show that this is no longer true in the non-abelian case. Note that Heitmann [30] has given an example of a commutative order with $D(\Lambda) = 0$ but without TFC.

Throughout this paper the term order will mean an order over a Dedekind ring R in a semisimple separable algebra over its quotient field K. Except for §1, I will also assume that K is a global field unless otherwise specified and will use the term "global order" to remind the reader of this assumption.

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[1] L. Levy has informed me that this result was obtained a few years ago (unpublished) by C. Odenthal who also showed that the Krull-Schmidt theorem holds for lattices over ZC_8.