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SURGERY ON THE EQUATORIAL IMMERSION I

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1. Introduction

Herein, smooth immersions of closed unoriented manifolds in codimension 1 Euclidean space are studied. The geometric topology of representative immersions as it relates to stable homotopy invariants is emphasized. The author's studies [1], [2], [3], [4], [5], and [6] are continued. Please see [2] and [6] for synopses.

(1.1) For each $k = 1, 2, \ldots, m$ there is an immersion

$$e: \bigcup_{j=1}^{k} S_j^{m-2} \to S^{m-1}$$

defined by the equation

$$e(x_1, \ldots, \hat{x}_j, \ldots, x_m) = (x_1, \ldots, x_{j-1}, 0, x_{j+1}, \ldots, x_m).$$

Here the domain is the disjoint union of k $(m-2)$-spheres,

$$S_j^{m-2} = \left\{(x_1, \ldots, \hat{x}_j, \ldots, x_m) : \sum x_k^2 = 1\right\};$$

$e|_{S^{m-2}}$ is an embedding, but the union is immersed. Such an immersion is called *an equatorial immersion* since each S_j is embedded as an equator of S^{m-1}. The multiple points of $(e, \bigcup_{j=1}^{k} S_j^{m-2})$ are spheres of lower dimensions. This immersion is null bordant since it is obtained by a piggy back sequence [6] of $(0,0), (0,1), \ldots, (0, k-1)$ surgeries on the empty immersion. Please recall a (j, r)-*surgery* attaches a hollow j-handle, $D^j \times S^{n-j}$, to an immersion $i: M \to \mathbf{R}^{n+1}$; the core disk, $D^j \times \{0\}$, lies in the r-tuple set $(0 \in D^{n+1-j})$.

The equatorial immersion is a prototype for piggy back sequences of surgeries in the following sense. If a piggy back sequence of $(j, 0), \ldots,$

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