ILLINOIS JOURNAL OF MATHEMATICS
Volume 32, Number 4, Winter 1988

REAL ALGEBRAIC CURVES AND COMPLETE INTERSECTIONS

BY

WOJCIECH KUCHARZ

1. Introduction

In this paper real algebraic varieties and real algebraic morphisms are understood in the sense of Serre [10] (Serre considers algebraic varieties over an algebraically closed field but his basic definitions make sense over any field). In particular, we do not assume that algebraic varieties are irreducible. An algebraic variety $(X, \mathcal{O}_X)$ will be simply denoted by X if no confusion is possible. The set of singular points of X will be denoted by $\operatorname{Sing}(X)$. We say that a family $\{Y_i\}_{i=1,\dots,k}$ of subvarieties (not necessarily closed) of X is in general position if for each point x in the union $Y_1 \cup \cdots \cup Y_k$, the family $\{T_x(Y_i)\}_{i\in\Lambda(x)}$, where $\Lambda(x) = \{i \mid x \in Y_i\}$, of vector subspaces of $T_x(X)$ ($T_x(\cdot)$ is the Zariski tangent space at x) is in general position, i.e.,

$$\operatorname{codim} \bigcap_{i\in\Lambda(x)} T_x(Y_i) = \sum_{i\in\Lambda(x)} \operatorname{codim} T_x(Y_i).$$

A subvariety Y of X will be called an algebraic hypersurface if Y is of pure codimension 1 in X. By a real algebraic curve in X we shall mean a subvariety of X of pure dimension 1.

Any real algebraic variety can be endowed with the strong topology induced from the Euclidean topology on the reals. Unless otherwise explicitly specified we shall always consider the *strong topology*. However, the terms "closed subvariety" or "open subvariety" refer to the Zariski topology.

DEFINITION 1.1. *A real algebraic variety X of dimension n is said to be admissible if there exists a sequence of real algebraic morphisms $\pi_i\colon X_i \to X_{i-1}$, $i = 1,\dots,k$, such that*

(i) *X_0 is an affine nonsingular real algebraic variety diffeomorphic, as a C^∞ manifold, to the unit n-dimensional sphere S^n,*

Received December 1, 1986.