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SOME GEOMETRIC CONSEQUENCES OF THE WEITZENBÖCK FORMULA ON RIEMANNIAN ALMOST-PRODUCT MANIFOLDS; WEAK-HARMONIC DISTRIBUTIONS

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0. Introduction

In this paper, we prove some geometric consequences obtained from certain linear relations among linear invariants of Riemannian almost-product manifolds. We also define and study weak-harmonic distributions.

In Section 1, we obtain a consequence of the Weitzenböck formula, (Theorem 1.2), which will be used in the next section.

Section 2 begins with general concepts on Riemannian almost-product manifolds.

A Riemannian almost-product manifold is a triplet $(\mathcal{M}, g, P)$, where $(\mathcal{M}, g)$ is a Riemannian manifold and P is a $(1,1)$-tensor field on $\mathcal{M}$ satisfying $P^2 = I$ and $g(PM, PN) = g(M, N)$, $M, N \in \mathfrak{X}(\mathcal{M})$. The eigenspaces of P corresponding to the eigenvalues 1 and -1, at each point, determine two distributions $\mathscr{V}$ and $\mathscr{H}$, respectively called vertical and horizontal.

Next, we get a linear relation among linear invariants of Riemannian almost-product manifolds, (Theorem 2.8), by using Theorem 1.2, from which we deduce some geometric consequences. Among these it is necessary to note that:

THEOREM. *A Riemannian almost-product manifold $(\mathcal{M}, g, P)$ with non-negative sectional curvature in which $\mathscr{V}$ and $\mathscr{H}$ are foliations whose mean curvatures, restricted to each horizontal and vertical leaf respectively, have zero divergence, is necessarily locally a product.*

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