

PRESENTATIONS OF MODULES WHEN IDEALS NEED NOT BE PRINCIPAL

BY

ROBERT M. GURALNICK AND LAWRENCE S. LEVY¹

**Dedicated to the memory of Irving Reiner, with gratitude for his caring
and encouragement**

We extend elementary divisor theory by studying presentations of modules over a class of rings that includes coordinate rings of affine curves, and the orders over Dedekind domains studied in integral representation theory. As an application, we answer a question of Nakayama about the uniqueness of the diagonal form of matrices over noncommutative principal ideal domains.

As a further application of our methods, we extend the Drozd Cancellation Theorem to modules over the rings we work with.

Let f and $g: P \rightarrow U$ be presentations of a module over a ring Λ . In other words, P is a projective module, and f and g are surjective Λ -module homomorphisms. (Modules are always finitely generated in this paper, unless otherwise stated.) As in matrix theory, we say that f is *equivalent* to g (notation: $f \sim g$) if there exist automorphisms α and β of P and U , respectively, such that $\beta f \alpha = g$.

Using this terminology, we can restate the main result of elementary divisor theory in the following form: If Λ is a commutative PID (principal ideal domain) and there is a presentation $P \rightarrow U$, then any two presentations of U by P are equivalent to each other. We say, more briefly, U is *uniquely presentable* by P .

To see the relation between this and more conventional statements of the elementary divisor theorem, let $g: \Lambda^n \rightarrow U$ be a presentation of an arbitrary Λ -module. We want to “diagonalize” g , that is, show that g is equivalent to a direct sum of presentations of cyclic modules. Assuming that we already know that U is a direct sum of cyclic modules, it is not hard to show that U is a direct sum of n cyclic modules. Say $U = U_1 \oplus \cdots \oplus U_n$. For each i there is a

Received July 12, 1987.

¹The authors were partially supported by grants from the National Science Foundation.