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OPERATORS INTERPOLATING BETWEEN RIESZ POTENTIALS AND MAXIMAL OPERATORS

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1. Introduction

Let λ be normalized Lebesgue measure on either the unit ball or the unit sphere in $\mathbf{R}^n$ and write λ_r for the dilate of λ defined by

$$\langle f, \lambda_r \rangle = \int_{\mathbf{R}^n} f(rx)\, d\lambda(x), \quad r > 0.$$

Suppose $1 \leqq p \leqq q \leqq \infty$, $1 \leqq s \leqq \infty$ and suppose f is continuous with compact support. When λ is the measure on the ball, define

$$S_{p,q,s} f(x) = \left[\int_0^\infty |r^{n/p - n/q} \lambda_r * f(x)|^s \frac{dr}{r} \right]^{1/s},$$

$$S_{p,q,\infty} f(x) = \sup_{r>0} r^{n/p-n/q} |\lambda_r * f(x)|.$$

When λ is the measure on the sphere, define operators $T_{p,q,s}$ and $T_{p,q,\infty}$ analogously. For nonnegative f, both $S_{p,q,1} f$ and $T_{p,q,1} f$ are multiples of the Riesz potential $I_\alpha(f)$ when $\alpha = n/p - n/q$. Hence $S_{p,q,1}$ and $T_{p,q,1}$ are bounded from $L^p (= L^p(\mathbf{R}^n))$ to L^q whenever $1 < p < q < \infty$. On the other hand, $S_{p,q,\infty}$ and $T_{p,q,\infty}$ are maximal operators, weighted to allow the possibility of $L^p - L^q$ boundedness. Indeed, $S_{p,p,\infty}$ is the Hardy-Littlewood maximal operator and therefore bounded on L^p for $1 < p \leqq \infty$, while $T_{p,p,\infty}$ is the spherical maximal operator, now known to be bounded on L^p when $n/(n-1) < p \leqq \infty$ (see [7], [2]). In general, and especially when $s = 2$, the functions $S_{p,q,s} f$ and $T_{p,q,s} f$ are reminiscent of g-functions. The purpose of this paper is to begin the study of the following question:

For what values of p, q, and s is $T_{p,q,s}$ bounded from L^p to L^q?

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