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CHARACTERIZATION OF BANACH SPACES OF CONTINUOUS VECTOR VALUED FUNCTIONS WITH THE WEAK BANACH-SAKS PROPERTY

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Introduction

A Banach space E is said to have the Banach-Saks property (resp. weak Banach-Saks property) if for every bounded sequence (resp. weakly convergent sequence) (x_n) in E, you can choose a subsequence (x'_n) of (x_n) such that the sequence

$$(y_n) = \left(\frac{x'_1 + \cdots + x'_n}{n}\right)$$

converges in the E-norm.

We shall refer to these properties as the B.S.P. and the W.B.S.P.

It is known that a Banach space E with the B.S.P. is reflexive. So, it is clear that a $C(K)$ space (being $C(K)$, the Banach space of the continuous functions from K to $\mathbf{R}$, and being K, a compact Hausdorff space) has the B.S.P. iff K is finite.

Much more interesting in this context of $C(K)$ spaces is the W.B.S.P. The following characterization of $C(K)$ spaces with the W.B.S.P. is due essentially to N. Farnum (see [2]).

THEOREM 1. *Let K be a compact Hausdorff space. Then $C(K)$ possesses the W.B.S.P. if and only if*

$$K^{(\omega)} = \bigcap_{n=1}^{\infty} K^{(n)} = \emptyset$$

where $K^{(0)} = K$ and $K^{(n)}$ is the set of all accumulation points of $K^{(n-1)}$ for $n \in \mathbf{N}$.

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