ILLINOIS JOURNAL OF MATHEMATICS
Volume 33, Number 2, Summer 1989

INFINITESIMAL RIGIDITY OF PRODUCTS OF SYMMETRIC SPACES

BY

JACQUES GASQUI AND HUBERT GOLDSCHMIDT

Let (X, g) be a compact symmetric space. We say that a 1-form or a symmetric 2-form on X satisfies the zero-energy condition if all its integrals over the closed geodesics of X vanish; an exact 1-form and the Lie derivative of the metric g along a vector field on X always satisfy the zero-energy condition. The space (X, g) is infinitesimally rigid if the only symmetric 2-forms on X satisfying the zero-energy condition are the Lie derivatives of the metric g.

In this paper, which is a sequel to [6], we investigate the infinitesimal rigidity of a product $X = Y \times Z$ of compact symmetric spaces Y and Z and generalize the results of [6] concerning the product $S^1 \times \mathbf{RP}^n$. We give a criterion for the infinitesimal rigidity of $Y \times Z$ mainly in terms of properties of Y and Z (Theorem 2.1) from which we deduce the infinitesimal rigidity of an arbitrary product $X_1 \times \cdots \times X_r$, where each X_j is either a projective space, different from a sphere, or a flat torus, or a complex quadric of dimension ≥ 5. This englobes all the previously known infinitesimal rigidity results (see [8]) and gives the first known examples of non-flat infinitesimally rigid symmetric spaces of arbitrary rank.

One of the main ingredients of our proofs is the characterization of exact 1-forms on these spaces in terms of closed geodesics. In [14] and [7], it is shown that the 1-forms on a projective space, which is not a sphere, satisfying the zero-energy condition are exact (see also [8]); the corresponding fact for flat tori is given by [13], and for complex quadrics of dimension ≥ 4 by [3].

We consider the product $X = Y \times Z$ and assume that Y and Z are infinitesimally rigid. We also suppose that the 1-forms on Y and Z which satisfy the zero-energy condition are exact. Let h be a symmetric 2-form on X satisfying the zero-energy condition. To prove that h is a Lie derivative of the metric, most of the methods and computations introduced in [6] to treat the case of $S^1 \times \mathbf{RP}^n$, with $n \geq 2$, are used here. Several important new features occur, especially because the dimensions of Y and Z may both be greater than one. We first wish to show that h is locally a Lie derivative of the metric by proving that it lies in the kernel of the differential operator Q_g of order 3 of [4], which is the compatibility condition for the Killing operator. The in-

Received April 23, 1987.