ILLINOIS JOURNAL OF MATHEMATICS
Volume 33, Number 2, Summer 1989

EXTREME OPERATORS ON FUNCTION SPACES

BY

STEPHEN T.L. CHOY

1. Introduction

Let A, B be Banach algebras and let i be the identity of A. The closed unit ball of A is denoted by $S(A)$ and the set of extreme points of $S(A)$ is denoted by ext $S(A)$. The closed unit ball of $\mathscr{L}[A, B]$, the Banach space of bounded linear operators from A to B, is denoted by $S[A, B]$. If X is a locally compact Hausdorff space let $C(X, A)$ stand for the space of continuous functions from X to A and let $C_0(X, A)$ stand for the subspace of continuous functions vanishing at infinity. Then $C_0(X, A)$ is a Banach algebra under the supremum norm. If $A = \mathbf{C}$, the set of all complex numbers, we simply write $C(X)$ and $C_0(X)$. The σ-algebra of Borel sets of X is denoted by $\mathscr{B}(X)$ and the set of bounded regular Borel measures is denoted by $M(X)$.

For bounded linear operator T: $C_0(X, A) \to B$ let m: $\mathscr{B}(X) \to \mathscr{L}[A, B^{**}]$ be its representing measure and let $|m|$, $\tilde{m}$ be its total variation and semivariation respectively, i.e.,

$$|m|(X) = \sup\left\{ \sum \|m(e_i)\| : \{e_i\} \in \pi(X) \right\},$$

$$\tilde{m}(X) = \sup\left\{ \left\| \sum m(e_i) x_i \right\| : \{e_i\} \in \pi(X),\, x_i \in S(A) \right\};$$

where $\pi(X)$ denotes the collection of all the (disjoint) finite Borel-partitions of X.

It is known that $\|T\| = \tilde{m}(X)$ and that if T is weakly compact then m: $\mathscr{B}(X) \to \mathscr{L}[A, B]$ (see [1], for example).

An extreme point of $S[A, B]$ is called an *extreme operator* from A to B. An operator T in $S[A, B]$ is a *nice operator* if $T^*[\text{ext } S(B^*)] \subset \text{ext } S(A^*)$, where T^* is the adjoint operator of T. It is known that every nice operator is an extreme operator and that the converse assertion is not true in general. Several authors studied the relationship between these two operators on various settings (see [2] and [9], for example). Extreme operators from function spaces

Received March 16, 1987.