ILLINOIS JOURNAL OF MATHEMATICS
Volume 33, Number 2, Summer 1989

THE MAXIMAL OPERATORS RELATED TO THE CALDERÓN-ZYGMUND METHOD OF ROTATIONS

BY

LUNG-KEE CHEN

1. Introduction and result

Let a_i, $i = 1, \dots, n$, be positive numbers, $0 < a_1 < a_2 < \cdots < a_n$. Define

$$\delta_t x = \left(t^{a_1}x_1, \dots, t^{a_n}x_n\right), \quad t > 0,$$

where $x = (x_1, \dots, x_n) \in R^n$. Let $\tau = a_1 + \cdots + a_n$. and v always denote a unit vector,

$$v = \left(v_1, \dots, v_n\right) \in S^{n-1},$$

and $d\sigma(v)$ denote the Lebesgue measure on S^{n-1}. Let $L^p(L^q(S^{n-1})R^n)$ denote mixed norm Lebesgue spaces. More precisely, if

$$\|g\|_{L^p(L^q)} = \left\| \|g\|_{L^q(S^{n-1})} \right\|_{L^p(R^n)} = \left[\int_{R^n}\left(\int_{S^{n-1}} |g(v, x)|^q \, d\sigma(v)\right)^{p/q} dx\right]^{1/p} < \infty,$$

then we say $g(v, x) \in L^p(L^q)$. Define

$$M_v f(x) = \sup_{r>0} \frac{1}{r} \int_0^r |f(x - \delta_t v)| \, dt.$$

R. Fefferman [2] proved that if $a = \cdots = a_n = 1$ then $M_v f$ is bounded on $L^p(L^2)$, for $p > 2n/n + 1$. Further developments are found in [1] and [3].
In this paper, we prove the following theorem.

THEOREM. *If $f \in L^p(R^n)$, then*

$$\|M_v f\|_{L^p(L^q)} \le C\|f\|_p,$$

Received April 6, 1987.