ILLINOIS JOURNAL OF MATHEMATICS
Volume 33, Number 2, Summer 1989

FIXED POINTS OF ISOMETRIES AT INFINITY IN HOMOGENEOUS SPACES

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Introduction

Let M be a simply connected homogeneous riemannian manifold of nonpositive curvature. Since M admits a simply transitive and solvable Lie group G of isometries, it can be represented as the Lie group G endowed with a left invariant metric of nonpositive curvature. If $\mathfrak{g}$ is the Lie algebra of G then $\mathfrak{g} = [\mathfrak{g}, \mathfrak{g}] \oplus \mathfrak{a}$ where $\mathfrak{a}$, the orthogonal complement of $[\mathfrak{g}, \mathfrak{g}]$ in $\mathfrak{g}$ with respect to the metric, is an abelian subalgebra of $\mathfrak{g}$.

In this paper, we describe the set of fixed points of G at infinity and we classify all isometries defined by elements of G when M has no de Rham flat factor; more precisely, we show that the elements of $[G, G]$ are parabolic and the hyperbolic elements of G are those conjugate to $\exp(\mathfrak{a})$.

In Section 1, we study the action of right invariant vector fields on the geodesics $\gamma_Z(t) = \exp tZ$ with $Z \in \mathfrak{a}$. All stable Jacobi fields on γ_Z are determined on certain regular elements Z of $\mathfrak{a}$ (Corollary 1.3). Section 2 is devoted to describe, for each Z in $\mathfrak{a}$, the subgroups of G that fix $\gamma_Z(\infty)$ (Corollary 2.6). In the third section, the set of fixed points of G at infinity is described (Theorem 3.4) and all isometries coming from left translations by elements of G are classified (Corollaries 3.7 and 3.9). In particular, if M is not a visibility manifold and $I(M)$ (or $I_0(M)$) has a fixed point at infinity (for instance if M is not symmetric) this point is necessarily a flat point at infinity (Corollary 3.5).

Finally, in Section 4 we summarize some results about the points at infinity that can be joined by a geodesic to a fixed point of G.

Preliminaries

Let M denote a complete and simply connected riemannian manifold of nonpositive curvature ($K \leq 0$). All geodesics in M are assumed to have unit

Received March 9, 1987.

[1]Partially supported by CONICOR.