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GENUS ACTIONS OF FINITE SIMPLE GROUPS

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1. Introduction

An action of a finite group G on a Riemann surface S is called a genus action provided G acts effectively and analytically on S but does not so act on any Riemann surface of lesser genus. The purpose of this paper is to prove:

THEOREM A. *Let G be a finite simple group (simple shall always mean simple nonabelian), $T(r, s, t)$ a Fuchsian triangle group, Δ a surface group, and S the closed Riemann surface induced from the short exact sequence*

$$1 \to \Delta \to T(r, s, t) \to G \to 1.$$

Then either

(i) *G is normal in* Aut S, *the full group of automorphisms of S, or*

(ii) *G is isomorphic to $L_2(7)$ and $(r, s, t) = (3, 3, 7)$.*

THEOREM B. *Let G be a finite simple $(2, s, t)$-group with genus action on the Riemann surface S arising from the short exact sequence*

$$1 \to \Delta \to \Gamma \to G \to 1.$$

Then G is normal in Aut S. *Moreover, if Γ is a triangle group, then* Aut S *embeds faithfully in* Aut G.

Remark. The requirement in Theorem B that G be $(2, s, t)$-generated is far less restrictive than appearances would at first indicate. Indeed it is a long-standing conjecture that *every* finite simple group is so generated. In particular, the conjecture has been verified for the families of alternating and sporadic groups, among others (see, for example, [1], [2], [3], [4], [8], [15]). Concerning the requirement that Γ be a triangle group, this appears to be the case with

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