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HILBERT-SCHMIDT INTERPOLATION IN CSL-ALGEBRAS

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One form of interpolation problem in operator algebras is the following: given vectors x and y in a Hilbert space $\mathscr{H}$ and an operator algebra $\mathscr{A}$ acting on $\mathscr{H}$, does there exist an operator T in $\mathscr{A}$ such that $Tx = y$? Furthermore, if interpolation is possible for a pair of vectors x and y, what is the minimum norm possible for an operator T which maps x to y? A variation of this problem asks for the interpolation of a linearly independent set of vectors $\{x_1, \dots, x_n\}$ onto a second set of vectors $\{y_1, \dots, y_n\}$.

An early and particularly deep example of this type of interpolation theorem is Kadison's Transitivity theorem [7]: if $\mathscr{A}$ is a C^*-algebra which acts irreducibly on $\mathscr{H}$, if $\{x_1, \dots, x_n\}$ is a linearly independent set of vectors in $\mathscr{H}$ and if $\{y_1, \dots, y_n\}$ is any set of vectors in $\mathscr{H}$, then there is an operator T in $\mathscr{A}$ such that $Tx_i = y_i$, for all i. Another example is the following theorem, which was first proven for nest algebras by Lance [9] and then extended to all CSL algebras in [5]: let Alg $\mathscr{L}$ be a reflexive operator algebra with a commutative subspace lattice $\mathscr{L}$. Let x and y be vectors in $\mathscr{H}$. Then there is an operator $T \in \operatorname{Alg} \mathscr{L}$ such that $Tx = y$ if, and only if,

$$\sup_{E \in \mathscr{L}} \frac{\|E^\perp y\|}{\|E^\perp x\|} < \infty.$$

If this supremum is finite, then it is the minimum norm for an interpolating operator for x and y. (A fraction with both numerator and denominator equal to 0 is taken to be 0.)

A recent paper by N.J. Munch [12] solves the interpolation problem in the setting of nest algebras subject to the additional restriction that the interpolating operator must be a Hilbert-Schmidt operator. As it happens, nest algebras are of interest in linear system theory; indeed, Munch interprets some of his results in terms of signal reconstruction. Some authors in system theory have found it convenient to go beyond Hilbert resolution space and the correspond-

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