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THE q-PARTS OF DEGREES OF BRAUER CHARACTERS OF SOLVABLE GROUPS[1]

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0. Introduction

All groups considered are finite and p and q denote primes. Assume $q \neq p$ and every $\varphi \in IBr_p(G)$ has q'-degree. In [11], we showed that if G is p-solvable, then G is in fact q-solvable with metabelian Sylow-q-subgroups. While, in general, G may not be q-solvable (e.g., $PSL(2, p)$ with $q = 2$), it remains open whether a Sylow-q-subgroup of G is necessarily metabelian. In Section 1 below, we assume that $q^{e+1} \nmid \varphi(1)$ for all $\varphi \in IBr_p(G)$ and give, for solvable G, a linear bound for both the derived length of a Sylow-q-subgroup of G and the q-length of G. In fact, if $N \trianglelefteq G$ and $\mu \in IBr_p(N)$, we bound the derived length of a Sylow-q-subgroup of G/N in terms of the largest power of q dividing $\varphi(1)/\mu(1)$ as φ varies over $IBr_p(G|\mu)$, the irreducible Brauer characters of G lying over μ.

Assume that $p^{e+1} \nmid \varphi(1)$ for all $\varphi \in IBr_p(G)$. If G is p-solvable, we give a linear bound for the p-rank of $G/O_p(G)$ and a logarithmic bound for the p-length of $G/O_p(G)$, but give no bound for the derived length of a Sylow-p-subgroup of $G/O_p(G)$. The methods here are different than for $q \neq p$ and we show that we cannot derive these bounds "locally," i.e., relative to a character of a normal subgroup. In closing, we do improve known bounds for the derived length of a Sylow-p-subgroup of p-solvable groups in terms of the degrees of ordinary characters.

All groups considered are finite. We let $l_p(H)$ and $r_p(H)$ denote the p-length and p-rank (respectively) of a p-solvable group H, i.e., $r_p(H)$ is the largest integer r such that p^r is the order of a p-chief factor of H. Also $dl_p(G0$ denotes the derived length of a Sylow-p-subgroup of G.

Section 1. $q \neq p$

In this section, for solvable G, we bound $dl_q(G)$ in terms of the largest power of q that divides the degree of some irreducible Brauer character of G.

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