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PERTURBATION THEORY IN DIFFERENTIAL HOMOLOGICAL ALGEBRA I

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1. Introduction and notation

1.1. Introduction. Differential homological algebra extends the classical machinery of homological algebra to differential algebras and modules. As first introduced by Eilenberg and Moore [5], the functor *Tor* (differential Tor) can be constructed in terms of resolutions relative to the category of differential modules, that is, in terms of *bicomplexes*. The category **DGA** of differential graded augmented algebras and differential graded augmented algebra maps was enlarged to the category **DASH** in [7]. **DASH** and **DGA** have the same objects; but **DASH**$(A, A') =$ **DC**$(\bar{B}A, \bar{B}A')$ where **DC** is the category of differential graded augmented coalgebras and $\bar{B}$ denotes the bar construction functor. The functoriality of *Tor* and *Cotor* was extended to this larger category and used to explain certain "collapse theorems" for the Eilenberg-Moore spectral sequence. These collapse theorems are subsumed by the theory in [6] which uses *multicomplexes*, a useful generalization of bicomplexes [22]. Perturbation theory has come to refer to a systematic way of constructing multicomplexes and the purpose of this paper is to begin a study of such algorithms.

DASH itself can be expanded to include more objects and this can be done in a way that generalizes results from [7]. More specifically, it is shown in [10] that if $A \in$ **DGA** is chain homotopy equivalent to a differential graded *module* M in a certain way (see 1.2 below) then M inherits an A_∞ structure and there is a differential graded coalgebra map which is a homology isomorphism from the "tilde construction" (which is what is meant by an "A_∞ structure" [20], [21]) of M to the bar construction of A. With strong conditions on a chain homotopy equivalence between an algebra A and another algebra M, an algorithm was given in [7, 4.1] for the construction of a chain homotopy equivalence which is a differential graded coalgebra map $\bar{B}M \to \bar{B}A$. In §3 we will show that the algorithm given in [10] reduces to this one under these special hypotheses. In other words, a special case of the proof in [10] is the proof of the special case in [7]. Next, we consider the algorithm presented in [8] and called "the basic perturbation lemma" in [14]. We show that the

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