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G^∞-FIBER HOMOTOPY EQUIVALENCE

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Introduction and preliminaries

Let G be a compact, connected Lie group, and V, W two complex G-modules. Denote the unit spheres by SV, SW. In this article we shall be concerned with maps over BG,

$$\begin{array}{ccc} EG \times_G (SV, SV^G) & \xrightarrow{f} & EG \times_G (SW, SW^G) \\ & \searrow \quad \swarrow & \\ & BG & \end{array}$$

where $EG \to BG$ is a universal G-bundle. Such maps are studied in [9]. It can be easily seen that they are exactly those induced by equivariant maps $EG \times SV \to SW$, i.e., by the so-called G^∞-maps $SV \to SW$. We shall say that f is a *G^∞-equivalence* if, and only if, f is the degree-one map on the fibers. Note that according to Dold's theorem [8] a G^∞-equivalence is a fiber-homotopy equivalence, and therefore it admits a G^∞-equivalence as an inverse. Also note that, in the equivariant case, the notion of a G^∞-equivalence is just the notion of quasi-equivalence introduced in [13]. We shall say that the G^∞-equivalence $SV \to SW$ is *special* if, and only if, it induces a T^∞-equivalence

$$(SV, SV^T, SV^G) \to (SW, SW^T, SW^G),$$

where $T \subset G$ is a maximal torus. It is easy to see that a degree-one G-map is special [11].

In this article we first study how V and W are related to each other, given that SV and SW are G^∞-equivalent. The answer is formulated in terms of an appropriate K-theoretic degree, with values in the completion $R(G)^\wedge$ of the representation ring, defined in the manner of [12] and [7] and denoted by $\deg_G f$. We shall say that $\deg_G f$ is *rational* if, and only if, it lies in $R(G)$. It will be shown in §2 below that $\deg_G f$ is rational if $V \cong W$ and f is a G^∞-equivalence, or if f is equivariant. However, the inverse of a degree-one

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