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COMPACT HANKEL OPERATORS ON THE BERGMAN SPACE

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1. Introduction

Let $\mathbf{D} = \{z \in \mathbf{C}: |z| < 1\}$ denote the open unit disk in the complex plane $\mathbf{C}$, and let A denote the usual Lebesgue area measure on $\mathbf{C}$. For $1 \le p < \infty$ and f: $\mathbf{D} \to \mathbf{C}$ Lebesgue measurable let $\|f\|_p = (\int_{\mathbf{D}}|f|^p\, dA/\pi)^{1/p}$. The Bergman space $L_a^p(\mathbf{D})$ is the Banach space of analytic functions f: $\mathbf{D} \to \mathbf{C}$ such that $\|f\|_p < \infty$. The Bergman space $L_a^2(\mathbf{D})$ is a Hilbert space; it is a closed subspace of the Hilbert space $L^2(\mathbf{D}, dA/\pi)$ with inner product given by

$$\langle f, g\rangle = \int_{\mathbf{D}} f(z)\overline{g(z)}\, dA(z)/\pi,$$

for $f, g \in L^2(\mathbf{D}, dA/\pi)$. Let P denote the orthogonal projection of $L^2(\mathbf{D}, dA/\pi)$ onto $L_a^2(\mathbf{D})$. The map $I - P$ is the orthogonal projection of $L^2(\mathbf{D}, dA/\pi)$ onto $L_a^2(\mathbf{D})^\perp$ (the orthogonal complement of $L_a^2(\mathbf{D})$ in $L^2(\mathbf{D}, dA/\pi)$). For a function $f \in L^\infty(\mathbf{D}, dA/\pi)$, the Hankel operator H_f: $L_a^2(\mathbf{D}) \to L_a^2(\mathbf{D})^\perp$ is defined by

$$H_f g = (I - P)(fg), \quad g \in L_a^2(\mathbf{D}).$$

It is clear that H_f is a bounded operator for every function $f \in L^\infty(\mathbf{D}, dA/\pi)$. In [2], Sheldon Axler raised the question of finding necessary and sufficient conditions on the function $f \in L^\infty(\mathbf{D}, dA/\pi)$ for the Hankel operator H_f to be compact. Sheldon Axler answered a special case of this problem in [3] where he considered conjugate analytic symbols. The "little Bloch" space $\mathscr{B}_0$ is the set of all analytic functions f on $\mathbf{D}$ for which

$$(1 - |z|^2)f'(z) \to 0 \quad \text{as } |z| \to 1^-.$$

Axler proved that for a function f in $L_a^2(\mathbf{D})$ (perhaps unbounded) the (densely defined) Hankel operator $H_{\bar f}$ is compact if and only if $f \in \mathscr{B}_0$. In [8], Kehe Zhu characterized the functions $f \in L^\infty(\mathbf{D}, dA/\pi)$ such that both Hankel operators H_f and $H_{\bar f}$ are compact. In this paper we will characterize the

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