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A SCALE OF LINEAR SPACES RELATED TO THE L_p SCALE

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1. Introduction and preliminary results

In [7] the Banach space $L_1(\mathbf{R}) \cap L_2(\mathbf{R})$ was investigated. In the present paper we consider a range of spaces of which $L_1(\mathbf{R}) \cap L_2(\mathbf{R})$ is one member. This scale of Banach spaces is closely related both to the L_p scale and to Hilbert space. Subspace structure and other linear topological properties of the scale are investigated.

Let (Ω, Σ, m) be a measure space and let $L_p(\Omega)$ be the usual Lebesgue space with the norm

$$\|f\|_p = \left(\int_\Omega |f|^p \, dm \right)^{1/p} \quad (0 < p < \infty)$$

and

$$\|f\|_\infty = \operatorname{ess\,sup}\left\{ |f(\omega)| \colon \omega \in \Omega \right\}.$$

For $0 < p \leq \infty$, let $Y_p(\Omega)$ be the collection of all measurable f such that

$$\|f\|_{Y_p} = \| f^* I(0,1) \|_p + \| f^* I(1,\infty) \|_2 < \infty$$

(here f^* denotes the decreasing rearrangement of $|f|$), and for $0 < n < \infty$ let $M_p(\Omega)$ consist of all f such that

$$\|f\|_{M_p} = \| f^* I(0,1) \|_2 + \| f^* I(1,\infty) \|_p < \infty$$

Finally, let $M_\infty(\Omega)$ be the closure of $L_2(\Omega)$ with respect to the norm

$$\|f\|_{M_\infty} = \| f^* I(0,1) \|_2.$$

Observe that if Ω is a probability space then

$$Y_p(\Omega) = L_p(\Omega) \quad \text{and} \quad M_p(\Omega) = L_2(\Omega),$$

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