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HARMONIC REFLECTIONS WITH RESPECT TO SUBMANIFOLDS

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1. Introduction

Geodesic symmetries on a Riemannian manifold (M, g) are local diffeomorphisms which play an important role in the treatment of the geometry of (M, g). Locally symmetric Riemannian manifolds are manifolds with *isometric* local geodesic symmetries. In [3] it is proved that "isometric" may be replaced by "*harmonic*" to characterize these spaces. Further, (local) reflections with respect to a curve are considered in [11] and the case of *harmonic reflections* has been studied in [1], [10].

All the results in these papers show that there is a strong relation between harmonic and isometric reflections. The main purpose of this paper is to clarify this relation. More precisely, the study of reflections with respect to a submanifold has been started in [2], [9]. In this paper we study harmonic reflections with respect to a submanifold and we will show that in the analytic case *a reflection with respect to a submanifold is harmonic if and only if it is an isometry*. As a corollary we obtain a result for *holomorphic* and *anti-holomorphic reflections* on a quasi-Kähler manifold.

2. Preliminaries

In this section we give a short description of the basic material we shall use in the rest of the paper. (See [6], [7] for more details.)

Let (M, g) be a Riemannian manifold of class C^∞ and B a (connected) topologically embedded submanifold which is relatively compact. Let $m \in B$ and let $\{E_1, \dots, E_n\}$, $n = \dim M$, be a local orthonormal frame field of (M, g) defined along B in a neighborhood of m. Let $q = \dim B$ and specialize the moving frame such that $E_1, \dots, E_q$ are tangent vector fields and $E_{q+1}, \dots, E_n$ are normal vector fields. Further, let $(y^1, \dots, y^q)$ be a system of coordinates in a neighborhood of m in B such that

$$\frac{\partial}{\partial y^i}(m) = E_i(m), \quad i = 1, \dots, q,$$

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