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HIGHER LOGARITHMS

BY

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Dedicated to the memory of Professor Kuo-Tsai Chen

"And finally, in an attempt to unify the entire subject into a coherent whole, difficulties of a different order are encountered, and some central unifying principle has still to be discovered." (Lewin, Polylogarithms and Associated Functions [L] p. xv.)

1. Introduction

Few mathematicians would disagree with the assertion that the logarithm is one of the most important functions in mathematics. During the nineteenth century an analogous function, the *dilogarithm*, was the subject of much research. First defined by Leibnitz in 1696, the dilogarithm was subsequently studied by Euler, Spence, Abel, Hill, Jonquière Kummer, Lindelöf, Lobachevsky, and many others [L]. Recently there has been a resurgence of interest in this remarkable function, due in large part to the

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