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INTERPOLATION SETS FOR LIPSCHITZ FUNCTIONS ON CURVES OF THE UNIT SPHERE

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Introduction and statement of results

Let B denote the unit ball in $\mathbf{C}^n$, and S its boundary. For $\alpha \in (0,1]$, $Lip_\alpha(B)$ will denote the space of holomorphic functions in B satisfying a Lipschitz condition of order α with respect to the Euclidean distance. For a closed set $I \subset \mathbf{R}$, and $0 < \alpha < 1$, $\Lambda_\alpha(I)$ will denote the space of Lipschitz functions on I, and $\Lambda_1(I)$ the Zygmund class.

We also consider the Koranyi pseudodistance $d(z,w) = |1 - \bar{z}w|$, for $z, w \in S$, where

$$\bar{z}w = \sum_{i=1}^{n} \bar{z}_i w_i.$$

This defines a pseudodistance only on S, but we will as well consider it when one of the two variables is on $\bar{B}$.

We will work with a simple (without intersections) periodic curve of class C^3 γ: $\mathbf{R} \to S$. With a suitable parametrization (arc-length plus a dilatation) we will suppose from now on that γ is 2π-periodic and that there exists $\lambda > 0$ such that for each t, $|\gamma'(t)|^2 = \lambda$. We will write $I = [-\pi, \pi]$ and $\Gamma = \gamma(I)$. We will not distinguish between $\gamma(t)$ and its corresponding parameter on I.

Related to γ we introduce the *index of transversality*, T: $I \to \mathbf{R}$, given by

$$-iT(x) = \overline{\gamma'(x)}\,\gamma(x), \quad x \in I. \tag{1}$$

Complex-tangential curves (i.e. $\gamma'(t) \in P_{\gamma(t)}$ where $P_{\gamma(t)}$ is the complex-tangential space at $\gamma(t)$) correspond to $T = 0$ and transverse curves to $|T(x)| \geq M$. We introduce the set E of *complex-tangential* points of Γ, given by

$$E = \gamma(\{x \in I/T(x) = 0\}) = \{\zeta \in I/T_\zeta\Gamma \subset P_\zeta\} \tag{2}$$

where $T_\zeta\Gamma$ is the tangent space of Γ at ζ.

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