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ON THE MODULUS OF ABSOLUTE CONTINUITY OF HOLOMORPHIC FUNCTIONS IN THE BALL

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Introduction

The starting point of this work is the result proved in our previous paper [1] according to which if f is a holomorphic function in the unit ball B^n of $\mathbf{C}^n$ such that $R^n f \in H^1$, i.e., if

$$(1) \qquad \sup_r \int_S |R^n f(r\zeta)|\, d\sigma(\zeta) < \infty$$

(where $R = \sum_j z_j \partial/\partial z_j$ denotes the radial derivative), then f is continuous up to the boundary and it is absolutely continuous along any smooth simple curve on the unit sphere S (a particular case had been previously proved by F. Beatrous in [2]). Two natural questions arise. The first is to obtain a relation between the modulus of continuity of $R^n f$ as a function in $L^1(S)$ and the modulus of absolute continuity of f in $\overline{B}$. The second is to find out which form does it take in this context the general principle first pointed out by E.M. Stein in [7] stating that holomorphic functions with some kind of boundary regularity are automatically twice as regular in the complex tangential directions.

In this paper we deal with these two questions (in section 1 and 2 respectively). Rather than relying on the results of [1] we carry over alternative proofs of the global continuity of f and its absolute continuity on curves that also give the desired extra information. In some sense the methods used here are more direct and elementary than those of [1] but as a counterpart they just work under certain restrictions of f and the type of curves being considered.

We will consider all curves parametrized by arc-length, i.e., $\langle \varphi'(t), \varphi(t) \rangle = 1$; we will also consider associated to $\varphi(t)$ the function $T(t)$, which we call its

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