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GENERALIZATION OF MYERS' THEOREM ON A CONTACT MANIFOLD

BY

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1. Introduction

In 1941, Myers [4] proved that a complete Riemannian manifold for which $\operatorname{Ric} \geq \delta > 0$, is compact. In 1981, Hasegawa and Seino [3] generalized Myers' theorem for a Sasakian manifold by proving that a complete Sasakian (normal contact metric) manifold for which $\operatorname{Ric} \geq -\delta > -2$, is compact. Actually their proof uses only that the structure is K-contact and not the full strength of the Sasakian condition. A *K-contact structure* is a contact metric structure such that the characteristic vector field of the contact structure is Killing.

Now a contact metric structure is K-contact if and only if all sectional curvatures of plane sections containing the characteristic vector field are equal to 1 (see e.g. [1], p. 65) and hence there is a lot of positive curvature involved in the problem from the outset. The question then arises for a general contact metric structure: Can we relax the condition that the sectional curvature $K(\xi, X)$ of any plane section containing the characteristic vector field ξ be equal to 1; even if we must increase $-\delta$ from near -2 to near 0 to compensate? In general, the notion of a contact metric structure is quite weak; in fact, the set of all such structures associated to a given contact structure is infinite dimensional. So we seemingly must assume some condition generalizing the K-contact structure, then we can study $K(X, \xi) \geq \varepsilon > \delta' \geq 0$ and $\operatorname{Ric} \geq -\delta > -2$ where δ' is a function of δ.

Let M denote a $(2n+1)$-dimensional contact metric manifold with structure tensors (φ, ξ, η, g); i.e., η is a globally defined contact form

$$\left(\eta \wedge (d\eta)^n \neq 0\right),$$

ξ its characteristic vector field $(d\eta(\xi, X) = 0,\ \eta(\xi) = 1)$, g a Riemannian metric, and φ a skew-symmetric field of endomorphisms satisfying

$$\varphi^2 = -I + \eta \otimes \xi, \qquad \eta(X) = g(X, \xi), \qquad (d\eta)(X, Y) = g(X, \varphi Y).$$

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